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A toy model for the AdS/CFT correspondence

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Abstract: We study the large-N gauged quantum mecchanics for a single hermitean matrix in the Harmonic oscillator potential well as a toy model for the AdS/CFT correspondence. We argue that the dual geometry should be a string in two dimensions with a curvature of stringy size. Even though the dual geometry is not weakly curved, one can still gain knowledge of the system from a detailed study of the open-closed string duality. We give a mapping between the basis of states made of traces (closed strings) and the eigenvalues of the matrix (D-brane picture) in terms of Schur polynomials. This is interpreted as an exact open-closed duality. We connect this model with a decoupling limit of $\mathcal{N} = 4$ SYM and the study of giant gravitons in $AdS_5 \times S^5$. We show that the two giant gravitons that expand along AdS_5 and S^5 can be interpreted in the matrix model as taking an eigenvalue from the Fermi sea and exciting it very much, or as making a hole in the Fermi sea respectively. This is similar to recent studies of the $c = 1$ string. This connection gives new insight on how to perform calculations for giant gravitons.

Keywords: D-branes, AdS-CFT and dS-CFT Correspondence, Matrix Models.

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1. Introduction

Recently the study of the $c = 1$ string theory has received a lot of attention, especially due to the work of McGreevy and Verlinde [1, 2], where a D-brane interpretation of the dual matrix model was discussed. Their work was partly based on Sen's observation that D-brane decay can be studied exactly in α' corrections [3] and the results of Fateev, Zamolodchikov and Zamolodchikov [4] on Liouville boundary field theory, where a description of the boundary states of the model was discussed. Many of these results have been made more precise in the literature [5, 6, 7], including a space-time interpretation in terms of type O strings for the solution to the non-perturbative instability of the original matrix model [8]. The $c = 1$ string corresponds to the large-N limit of matrices on the inverted (upside down) harmonic oscillator. A review of the early literature can be found in [9], and a more recent review can be found in [10]. The model can be nicely described in terms of the eigenvalues of a hermitean matrix X, which become fermions in the quantum theory [11]. The double scaling limit that gives rise to the $c = 1$ model is done in such a way that the Fermi level is close to the top of the potential, so that one can focus on the physics of the top of the hill. For this model the spectrum is continuous, and the observables of the model can be interpreted as giving rise to an S-matrix. The stringy states correspond to small ripples on the Fermi sea which scatter from the top of the hill and go back to infinity, while the dual geometry is a two dimensional string with a linear dilaton background. The matrix model is exactly solvable, so this matrix model provides a holographic description of quantum geometry on a space-time which has an asymptotically flat region. The interpretation in terms of D-branes means that the model is gauged, so one only considers gauge invariant states, which depend only on the eigenvalues of the matrix X. A lot of insight can be gained from studying the model in the phase space of the eigenvalues of the matrices, especially since one can give a very pictorial description of the model.

An equally solvable model, is the study of the large-N limit of the gauged ordinary harmonic oscillator. In general we can choose a more complicated potential, where the lagrangian is given by

$$L = N \int dt \,\mathrm{tr} \left[\frac{1}{2}(D_t X)^2 - V(X)\right] \tag{1.1}$$

where V is an arbitrary potential. For general $V(x)$ it is not possible to solve for the energy levels exactly, so we will concentrate on a particularly simple solvable model, where one can solve the system explicitly in more than one basis. Moreover, we will later show that the harmonic oscillator potential is special also because it appears as a decoupling limit of the $\mathcal{N} = 4$ SYM theory. This feature makes it clear that this one particular potential originates from a bona-fide string theory in ten dimensions, and may be interpreted as a string theory in its own right.

For the quadratic potential the only tunable parameter is N, so the effective expansion in planar diagrams is the ordinary 't Hooft expansion [12] and the string coupling constant is $1/N^2$. The expansion in this case does not affect the energies of the states, but it does affect their overlaps. Usually only theories in a double scaling limit are considered as string theories, but then, it is not usually assumed that the $\mathrm{U}(N)$ symmetry is gauged. In spite of the fact that the theory seems to be free, one can try to give a string theory interpretation of the model. This might turn out to be very topological in the end, as one does not fill the holes of Riemann surfaces with interactions. At the moment I do not have a good description of how to interpret this $1/N$ expansion in terms of a string worldsheet theory. In this paper this issue will not be explored. We will just trust that the $1/N$ expansion of 't Hooft always has such an interpretation.

A prominent feature of this model is that it has a discrete spectrum. In light of this fact, if this were to be interpreted as the holographic dual on some geometry, then the discreteness of the spectrum of states resembles the spectrum of dimensions of local operators in a conformal field theory, and should be viewed as giving the holographic dual of an AdS-like space-time in global coordinates. Indeed, we will interpret this model as an example of the AdS/CFT correspondence [13, 14, 15] which can be solved exactly. This has been proposed before as a toy model for AdS/CFT [26], but the proposal was not elaborated upon. The study of free fields as a route to AdS has also been explored in [16, 17], although there the purpose was to write the perturbation theory in Feynman diagrams so that it resembles propagation of fields in AdS. Here instead we take the free model as describing the dual AdS geometry.

From the spectrum of the theory one sees that the theory does not have a Hagedorn growth of states. At weak coupling and weak curvature this implies that the target space dimension is less than or equal to two. Given that we have a time variable on the boundary, holographic reasoning tells us that we should at least include one more dimension (the radial direction on AdS), so that the target space of the string would have at least two dimensions. These arguments point in opposite directions and single the target space dimension of the string theory as being equal to two.

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If we assume that the description resembles the behavior of an AdS_2 spacetime close to the conformal boundary of the associated spacetime, and that the dilaton has some asymptotic value which is fixed by the boundary conditions, then to saturate the string beta functions and to obtain a critical string, the curvature of the spacetime needs to be of order α'. This will cancel the contribution to the dilaton tadpole due to the non-critical dimension with the curvature of the embedding geometry. Strominger has also proposed a matrix model for AdS_2 [18] where similar features have been discussed. See also [19]. This precludes a straightforward geometric interpretation, as there are no regions in the geometry which are weakly curved compared to the string scale, were a semiclassical analysis would help us resolve geometry. Because of this issue, the dimension of the target space for the string can not be determined for certain. This matrix model has appeared before in the study of two dimensional black holes [20]. There, Ho has argued that the matrix quantum mechanics described above is a limit $M \to 0$ of a two dimensional black hole and it is related to the standard $c = 1$ matrix model. This seems difficult to achieve if we want to insist on keeping N finite but large. We will leave the target space geometric interpretation of the model as an open problem.

So, even if we don't understand the target space geometry, we can anyway study the system and gain insight from other points of view. We will try to understand the open-closed duality in as much detail as possible. After all, this is one way to understand the AdS/CFT correspondence when the spacetime geometry is highly curved [21]. In this sense, the gauged matrix harmonic oscillator should be a perfectly good toy model for the AdS/CFT.

The objective of the paper is to explain features of the AdS/CFT correspondence that can be realized in this matrix model, even in the absence of a string theory dual geometry. We find various ways to describe the spectrum of the matrix model exactly and relate them to each other, so that we have an open-closed duality in the system where all calculations can be performed. We also relate the model to $\mathcal{N} = 4$ SYM as a decoupled sector that describes half-BPS states of the theory. With this identification we can relate BPS states in the SYM theory and states in the matrix model. In particular, we find that the half-BPS D-branes in SYM theory (giant gravitons) can be identified with particular configurations in the matrix model which in examples of the $c = 1$ matrix model would also be called D-branes.

The plan of the paper is as follows. In section 2 we describe the spectrum of the matrix model in terms of "closed string" states. This is, in terms of single trace operators, in the spirit of [14]. Next, in section 3 we describe the spectrum in terms of the eigenvalues of X. We will call this picture the D-brane picture. In section 4 we describe a new basis for the closed strings in terms of Schur polynomials. This description follows from the work [26] where all the combinatorial description is laid out in detail. Here it is shown that these Schur polynomials capture the dynamics of the eigenvalues directly. We give a sketch of a proof by comparing the wave functions of these states in a particularly simple regime.

In section 5 we describe how this model relates to $\mathcal{N} = 4$ SYM theory as a decoupled sector of $\mathcal{N} = 4$ SYM and we find applications of this new correspondence to the study of giant gravitons in AdS space. We find that the two giant gravitons expanding into AdS_5

and S^5 correspond in the matrix model to taking an eigenvalue from the top of the Fermi sea of eigenvalues and exciting it by a large amount so that it is resolved from the Fermi surface, while the other giant graviton expanding into S^5 translates to making a hole state deep in the Fermi sea of eigenvalues. This behavior is exactly the same as the description of D-branes in the $c = 1$ matrix model and goes a long way to explain why the corresponding operators in SYM theory behave as D-branes.

In section 6 we describe other interesting physics related to this matrix model, and in particular we give a matrix model description of why the correct non-planar perturbation parameter in the plane wave limit scales as J^2/N, where J is the R-charge of a state.

Finally, we review some of the results and conclude.

2. Matrix description of the spectrum

The model we are studying is the large-N gauged harmonic oscillator. The theory can be solved by first solving the full matrix model theory and then imposing the gauge invariance of the states. This is what we will do in the following.

For reasons which will become apparent later, we will call this picture of the dynamics the *closed string* picture.

The system consists of a hermitean $N \times N$ matrix X, (or with explicit $\mathrm{U}(N)$ indices X^i_j) with potential $\frac{1}{2}\operatorname{tr}(X^2)$, and kinetic term $\frac{1}{2}\operatorname{tr}(D_t X)^2$, where

$$D_t(X) = \dot{X} + [A, X]$$

and A is the gauge connection and acts as a lagrange multiplier (which is also a hermitian $N \times N$ matrix). When $A = 0$, the system reduces to a collection of N^2 free harmonic oscillators, and we write the hamiltonian for these in terms of creation and annihilation operators

$$H = \frac{1}{2}(a^\dagger)^i_j a^j_i + \frac{1}{2}N^2 = \frac{1}{2}\operatorname{tr}(a^\dagger a) + \frac{1}{2}N^2 \tag{2.1}$$

where we have left the zero point energy of the system included.

Fixing $A = 0$ is a gauge choice. The only remnant of the gauge choice is that we have to satisfy the equations of motion of A, this is, $\delta L/\delta A = Q = 0$, and Q is the charge that generates gauge transformations. Thus we need to impose on the spectrum of states the gauge invariance constraint $Q = 0$.

The only non-trivial commutation relation of the a, $a^\dagger$ can be written as

$$[(a^\dagger)^i_j, a^k_l] = \delta^i_l \delta^k_j\,. \tag{2.2}$$

All other commutators between $a, a^\dagger$ vanish.

The vacuum is the unique state satisfying $a^j_i|0 >= 0$ for all i, j. This state is invariant under $\mathrm{U}(N)$ transformations by adjoint action on X.

An excited state of the system (ignoring the gauge constraint) is given by applying an arbitrary number of matrix creation operators to the vacuum. Each such operator increases the energy of the state by one.

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Now we want to impose the gauge constraint on the system. Each creation operator has one upper U(N) index and one lower U(N) index. If we act with k such operators on the vacuum we have a state which transforms as a tensor with k upper U(N) indices and k lower U(N) indices. To make a gauge invariant state, we need to contract the tensor indices of these states with an appropriate invariant tensor of U(N). These invariant tensors have to be formed by different possible orderings of δ^μ_ν, which contract all the upper indices with all the lower indices.

The collection of states obtained this way is the set of gauge invariant states in the large-N harmonic oscillator, and these are the physical states of the theory.

Starting with one creation operator we can follow the contraction of indices and write them like matrix multiplication. The states are then going to be given by products of expressions of the form

$$(a^\dagger)^{i_i}_{i_2}(a^\dagger)^{i_2}_{i_3}\dots(a^\dagger)^{i_n}_{i_1} \sim \mathrm{tr}((a^\dagger)^n)\,. \tag{2.3}$$

These single trace states are identified with closed string states in the AdS/CFT correspondence [14], so we will call these operators the closed string states. We will label them by their energy n. The operator that creates one closed string state of energy n is then

$$\beta^\dagger_n = A_n\,\mathrm{tr}((a^\dagger)^n) \tag{2.4}$$

where A_n is an appropriate normalization factor. For n fixed, and in the large-N limit $A \sim N^{-n/2}$.

The normalization is found by studying the norm of the state as follows

$$|\,\mathrm{tr}((a^\dagger)^n|0\rangle|^2 = \langle 0|\,\mathrm{tr}(a^n)\,\mathrm{tr}((a^\dagger)^n)|0\rangle \tag{2.5}$$

and this can be calculated by using Wicks theorem (free field contractions). Explicit results for the appropriate normalizations have been found to all orders in $1/N$ for all n in [22] in which they needed explicit expressions to understand the light cone string theory in the plane wave geometry [23]. See also [24, 25] for related calculations.

One can create multi string states by acting with various of these oscillators in succession. It is clear that $[\beta^\dagger_n, \beta^\dagger_m] = 0$, so the spectrum of the theory resembles a Fock space of states, where there is one closed string oscillator per positive integer $n > 0$. It is a well known but non-trivial fact that states with different "closed string" occupation numbers are approximately orthogonal in the large-N limit, so long as we keep the energy finite when we take the $N \to \infty$ limit. A lot of the detailed $1/N$ expansions for normalizations of the states and overlaps can be copied verbatim from the study of 1/2 BPS operators in the $\mathcal{N} = 4$ SYM, and we will return to this issue later in the paper.

Given these states, we can always order the string states in descending order, so that a multi-string state

$$|n_1, n_2, \dots, n_k\rangle = \beta^\dagger_{n_1}\dots\beta^\dagger_{n_k}|0\rangle \tag{2.6}$$

satisfies $n_1 \geq n_2 \geq n_3 \dots \geq n_k$. The total energy of the state above the ground state energy $\frac{1}{2}N^2$ is $\sum n_k = n$. The number of states with energy n is given by the partitions of n into positive integers for large-N. At finite N one needs to remember that traces of different length are not algebraically independent, indeed $tr((a^\dagger)^{N+1})$ can be written as a polynomial of traces of lower length.

In the large-N limit, the spectrum constructed above coincides with the spectrum of a chiral boson in $1+1$ dimensions. This point of view agrees with our description of the target space geometry in the introduction. The model suggests that the dual target space geometry has one field theory degree of freedom (this would be the "tachyon", as dilatonic gravity has no propagating degrees of freedom in two dimensions).

At finite N, the spectrum gets cut (this is called the stringy exclusion principle, which is non-perturbative in N), and the spectrum is determined by partitions of n into integers smaller or equal to N. To each configuration of traces we can associate a Young tableaux. We first order the integers in the multi-trace state so that they are decreasing. This is, we label the state

$$\beta^\dagger_{n_1}\dots\beta^\dagger_{n_k}|0\rangle \tag{2.7}$$

with $N \geq n_1 \geq n_2 \dots \geq n_k$ by a Young tableaux where the first column has n_1 boxes, the second column has n_2 boxes, etc. And the maximum length of each column is N.

3. The eigenvalue basis

Now, we will look at a second gauge choice, where we choose the matrix X to be diagonal. In the $c=1$ matrix model the eigenvalues represent D-branes. Here, we will use this same interpretation, so a description in terms of eigenvalues will be the description in terms of D-branes. This will be an *open string* description of the system.

Let us label the eigenvalues of X as λ_i. Then, when we write wave functions for the Schrödinger equation, they will be functions of λ_i. There is a discrete subgroup of $\mathrm{U}(N)$ which leaves the matrix X diagonal. This is the permutation group of the eigenvalues, so the wave functions have to be invariant under this symmetry, and this means that we get totally symmetric wave functions on the eigenvalues.

Classically, the lagrangian for the eigenvalue basis becomes

$$L = \sum \frac{1}{2}\dot\lambda_i^2 - \frac{1}{2}\lambda_i^2\,. \tag{3.1}$$

So the classical motion of the eigenvalues is that of a harmonic oscillator. However, quantum mechanically there is a change of measure from the matrix basis to the eigenvalue basis. This change of measure is the volume of the gauge orbit of the matrix X, and it is equal to the square of the Van der Monde determinant of the λ_i, namely

$$\mu = \Delta(\lambda)^2 = \prod_{i\neq j}(\lambda_i - \lambda_j)\,. \tag{3.2}$$

So that the Hamitonian in the quantum theory will be given by

$$H\psi = \frac{1}{2}\sum -\mu^{-1}\partial_{\lambda_i}(\mu\partial_{\lambda_i}\psi) + \lambda_i^2\psi \tag{3.3}$$

with ψ the wave function of the eigenvalues.

The measure can be absorbed in the wave functions for the λ_i, by attaching a factor of the Van der Monde to the wave function. We define $\psi(\lambda) = \Delta^{-1}(\lambda)\tilde\psi(\lambda)$, where $\tilde\psi(\lambda)$

is the new wave function in the X variables expressed in terms of the eigenvalues of X (these are the λ_i), and the measure for $\tilde{\psi}$ is is just $\prod d\lambda_i$. This can be done for any one matrix model quantum mechanics [11] with a single trace potential. This is a similarity transformation on the space of wave functions, so it affects the form of the Hamilltonian. The new hamiltonian is

$$\tilde{H} = \frac{1}{2}\sum_i -\partial^2_{\lambda_i} + \lambda_i^2 \tag{3.4}$$

so it becomes a hamiltonian for N free particles in the harmonic oscillator potential well. After this is done the wave functions are completely antisymmetric in the λ_i: the eigenvalues become fermions due to the Van Der Monde determinant. The system is reduced to N free fermions in a given potential, which for us is just $V(x) = x^2/2$. For our setup, an orthogonal basis for the N-particle wave functions is given by Slater determinants of one particle wave functions for the Harmonic oscillator (these are in turn given by Hermite polynomials times a gaussian factor $H^{n_k}(\lambda)\exp -\lambda^2/2$). This basis for the wave functions is given explicitly by

$$\psi(n_1, \dots, n_N) \sim \det\begin{pmatrix} H^{n_1}(\lambda_1) & H^{n_1}(\lambda_2) & \dots & H^{n_1}(\lambda_N) \\ H^{n_2}(\lambda_1) & H^{n_2}(\lambda_2) & \dots & H^{n_2}(\lambda_N) \\ \vdots & \vdots & \ddots & \vdots \\ H^{n_N}(\lambda_1) & H^{n_N}(\lambda_2) & \dots & H^{n_N}(\lambda_N) \end{pmatrix} \exp\left(-\sum \frac{\lambda^2}{2}\right). \tag{3.5}$$

In particular, the Fermi statistics imply that all of the n_k are different, and that we can order the n_i so that $n_1 > n_2 > n_3 > \dots > n_N \geq 0$. The energy of a state is then $\sum_i (n_i + 1/2)$. The ground state of the system is such that the n_i are minimal. This is, $n_{N-k} = k$. From here it follows that the ground state energy of the system is

$$\sum_{k=0}^{N-1} \frac{1}{2}(2k+1) = \frac{N^2}{2} \tag{3.6}$$

which coincides exactly with the c-number term in equation 2.1, where the hamiltonian is written in normal ordered form. We can also write the spectrum as the list of non-increasing integers given by $n'_1 = n_1 - N \geq n'_2 = n_2 + 1 - N \geq \dots \geq n'_k = n_k - (N-k) \geq n'_N = n_N \geq 0$. This coincides with the description of the spectrum given in terms of 'closed strings' in the previous section. The difference, however, is that in the eigenvalue basis, for different values of the list of integers n_α we get orthogonal states. Because of the description in terms of Fermions, the ground state describes a Fermi sea of eigenvalues, where the level of the sea is determined by N. We can look at the spectrum of excitations as given by exciting the Fermi surface of the Fermi sea. The highest fermion of the Fermi surface has it's energy raised by n'_1 units, the next to highest fermion has it's energy raised by n'_2 units and so on. For finite energy excitations (in the large N limit) only the topmost eigenvalues get excited beyond their ground states. One can also look at states whose energy scales with N in some way in the large-N limit. This is not anymore the usual large N limit of 't Hooft [12].[1]

[1]These types of states will become important later. The energy of the states we will look at will scale

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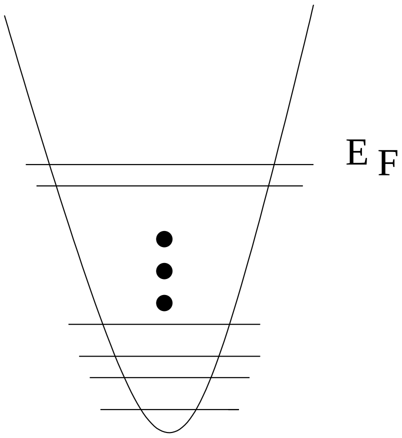

Figure 1: Filling the potential well.

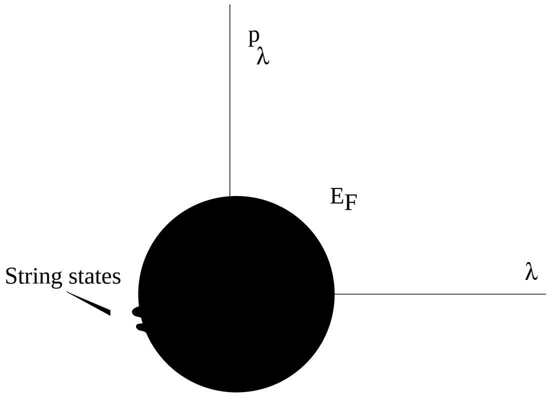

Figure 2: String states as small perturbations of the Fermi surface.

Again, one can describe a state in the spectrum by drawing a Young tableaux. The tableaux is written so that the first row has n'_1 boxes, the second row has n'_2 boxes, etc. The tableaux has only N rows, as there are only N different eigenvalues.

The following two pictures figures 1 and 2 illustrate how one fills the Harmonic oscillator potential well with fermions, and the description of the Fermi surface in the phase space of a single eigenvalue. The string states found in the previous section will be general small perturbations of the Fermi surface of the eigenvalue distribution in the the phase space of a single eigenvalue. The description in terms of closed strings describe collective excitations of the fermions, since the basis of states is different than the one found here, so they describe mixed states of fixed energy where we can not determine the energy of a single eigenvalue exactly. These collective excitations are interpreted as changes in the shape of an incompressible Fermi liquid droplet in the eigenvalue phase space. The incompressibility arises because each eigenvalue state occupies one quantum of area, and Fermi statistics forces the eigenvalues to be centered around different locations. A single closed string state with energy k is interpreted as a single quantum of a wave on the edge of the droplet with wave number k.

4. Schur polynomial basis

We will now use a third description of the spectrum of the theory. This again proceeds through choosing $A = 0$, but we will write the multi-string states in a different basis.

The main idea is to use the construction of gauge invariant states proposed in [26] based on Schur polynomials. The basis construction proceeds by writing an auxiliary space V which transforms in the fundamental of $\mathrm{U}(N)$. We can then think of a hermitian matrix X as a linear map $X : V \to V$. The character of X in V is exactly $\mathrm{tr}(X)$, and this is invariant under general changes of basis of V, which are done by the complexification of

proportionately to $N \sim g_s^{-1} = 1/((1/N))$, so they have the potential to be interpreted as D-branes, because their tension will be proportional to the inverse of the 't Hooft string coupling constant $g_s^2 = N^{-2}$.

U(N), namely $GL(N,\mathbb{C})$. Now, let us consider the tensor product space $\Omega_n = V^{\otimes n}$, and define an action of X on Ω_n as follows

$$X(v_1 \otimes \cdots \otimes v_n) = (Xv_1) \otimes (Xv_2) \otimes \cdots \otimes (Xv_n)\,. \tag{4.1}$$

If X is invertible, we can think of X as an element of $GL(N,\mathbb{C})$, and this is the group action of X on the tensor product. If we decompose Ω_n into irreducible representations of $GL(N,\mathbb{C})$, then X acts diagonally under this decomposition. The action of X commutes with the permutations of the vectors v_i, so it is possible to digonalize X and the permutation group simultaneously. Thus we decompose Ω_n in terms of representations of the permutation group of n elements. This sets up a correspondence between representations of the symmetric group of n elements and irreducible representations of $GL(N,\mathbb{C})$, which is described exactly by Young tableaux with n boxes. Thus we can associate to each Young tableaux a group representation of $GL(N,\mathbb{C})$ which sits inside Ω_n, and an associated action of X on that same representation which is induced from projection of the action of X on Ω_n. Said more simply, if we restrict to tensors in Ω_n of a specific symmetry type, then the action of X on these tensors induced from the action of Ω_n preserves the symmetry type.

Symmetry types of tensors are in one to one correspondence with representations R of SU(N). In each of these representations we can find a gauge invariant observable which is the character of X in the associated representation $\mathrm{tr}_R(X)$. This is very similar to the characterization of observables in two dimensional QCD in terms of Wilson loops around non-contractible cycles, taking all possible representations of the group into account [27]. See also [28]. With proper normalization, these are called Schur polynomials. We can extend this action to matrix valued operators acting on some Hilbert space, so we can use the following basis

$$\mathrm{tr}_R(a^\dagger)|0\rangle \tag{4.2}$$

as a collection of states of the large-N harmonic oscillator. At energy n over the ground state, there are as many partitions of n with less than or equal to N rows as there are Young Tableaux representing irreducible representations of SU(N). Moreover, as discussed in [26], these states are actually orthogonal, so one can build this way an orthonormal basis of states which capture all of the states of the gauged harmonic oscillator with $N \times N$ matrices.

Now, we want to ask what is the relation between the three basis of states we have discussed: the *closed string basis*, the eigenvalue basis (we called this an *open string* description as it related to D-branes), and this new basis which we will call the Schur polynomial basis.

Going from the string basis to the Schur polynomial basis is straightforward, as the projections to the different irreducible components of Ω_n are done by taking appropriate symmetrizations over rows of the Young tableaux, and antisymmetrizing over the columns. For example, we can take the antisymmetric and symmetric representations (with two boxes) and we find that

$$\mathrm{tr}_A(X) = \frac{1}{2}(\mathrm{tr}(X)^2 - \mathrm{tr}(X^2)) \qquad \mathrm{tr}_S(X) = \frac{1}{2}(\mathrm{tr}(X)^2 + \mathrm{tr}(X^2))\,. \tag{4.3}$$

The states $\mathrm{tr}_A(a^\dagger)$ and $\mathrm{tr}_S(a^\dagger)$ mix maximally the different number of traces, so these are always interpreted as multi-closed string states.

The surprise is that the Schur polynomial basis seems to coincide exactly with the eigenvalue basis. This equivalence of basis was hinted in [29, 26], and was also found in [30] in the study of the Calogero model. A sketch of the proof goes as follows.

Let us consider the wave functions in the eigenvalue basis. As we said previously, these are determined by Slater determinants of Hermite polynomials times a gaussian factor (which is common for all wave functions). Let us strip the gaussian part of the wave function, so we are left with polynomials of the eigenvalues only. Take the limit $\lambda_1 \gg \lambda_2 \gg \cdots \gg \lambda_N \gg 1$. In this limit the wave function is dominated by the leading term (up to normalization factors)

$$\psi(\lambda_1, \ldots, \lambda_N) \sim \lambda_1^{n_1} \ldots \lambda_N^{n_N} \tag{4.4}$$

with $n_1 > n_2 \cdots > n_N$.

Now let us consider the operators $(a^\dagger)^{\otimes n}|0\rangle \sim (X + \partial_X)^{\otimes n}|0\rangle$. The leading term in X for large X will be given by letting ∂_X act on the gaussian factor, so that we can approximate $(a^\dagger)^{\otimes n}$ by $X^{\otimes n}$ up to numerical factors.

Let us now look at the matrix X. We can diagonalize it and evaluate $\mathrm{tr}_R(X)$ explicitly. We do this by choosing X to be diagonal with eigenvalues chosen in decreasing order, in the same asymptotic regime that we chose in the eigenvalue basis. Choose the Cartan of $GL(N, C)$ so that X is in the Cartan. The highest weight state $|\alpha_R\rangle$ will have weights $n_1^R e_1 + n_2^R e_2 + \ldots$, where the e_i are the positive roots of $\mathrm{SU}(N)$ and the n_i^R are the lengths of the rows of the Young tableaux. The character of X will sum over the elements of the weight lattice that belong to the representation R. The leading term is X acting on the highest weight state, so that

$$\mathrm{tr}_R(X) \sim \lambda_1^{n_R^1} \ldots \lambda_N^{n_R^N} \tag{4.5}$$

we also need to remember that the ground state wave function in the eigenvalue basis has an extra leading term from the Van Der Monde determinant, $\lambda_1^{N-1} \ldots \lambda_{N-1}$. Multiplying both of these and stripping the gaussian term we find the asymptotic behavior

$$\mathrm{tr}_R(X)|0\rangle \sim \lambda_1^{n_R^1 + N} \ldots \lambda_N^{n_R^N} \tag{4.6}$$

with $n'_1 = n_R^1 + N > n'_2 = n_R^2 + (N-1) > \cdots > n'_N = n_R^N$.

We need to compare the asymptotic behavior we just found with the one coming from the Slater determinant wave functions in the eigenvalue basis from equation 4.4. We notice that the two states which are associated with the same Young tableaux, have the same asymptotic behavior.

Now, we order the states according to how fast they grow in the asymptotic regime we are studying. A monomial of the form $\lambda_1^{s_1} \ldots \lambda_N^{s_N}$ will have higher ordering than $\lambda_1^{t_1} \ldots \lambda_N^{t_N}$ if it is of higher degree: $\sum s_i \geq \sum t_i$. In the case of equality (which will be the case for the highest degree of the wave function), we also require that the smallest integer k for which $s_k \neq t_k$ is such that $s_k > t_k$. This ordering of the monomials (from the smallest to the highest) produces a filtration of the Hilbert space of energy eigenvalues of the same energy, which is tied to the asymptotic growth of the wave function. A filtration is a collection of subvector spaces $V_1 \subset V_2 \subset V_3 \subset \cdots \subset H$. Here the V_i are ordered by the asymptotic behavior of the most divergent term in the wave function. A wave function $\psi \in V_i$ is such that ψ diverges at most as fast as the associated monomial associated to V_i. The vector space quotients satisfy $\dim(V_{i+1}/V_i) = 1$. This ordering of monomials translates to an ordering of the Young tableaux, so that V_k is a vector space of dimension k generated by the k smallest young tableaux with a fixed number of boxes.

The fact that the two basis of states, associated to Schur polynomials and Slater determinants (lets us call them $E_1, \ldots, E_k$ and $E'_1, \ldots, E'_k$) have the same asymptotic behavior means that

$$V_i = \mathrm{span}(E_1, \ldots E_i) = \mathrm{span}(E'_1, \ldots E'_i) \tag{4.7}$$

and in particular, we get that since $\dim(V_1) = 1$, then $E_1 = E'_1$ (up to normalization). Also, the E_α and E'_α are an orthogonal basis for each V_i, so that $V_i = V_{i-1} \oplus \mathrm{span}(E_i) = V_{i-1} \oplus \mathrm{span}(E'_i)$ where the decomposition is in terms of orthogonal complements. It follows that $\mathrm{span}(E_i) = \mathrm{span}(E'_i)$, and since this is a one-dimensional vector space, we have that $E_i = E'_i$ (up to normalization). This shows that the two basis of states are equivalent.

In essence, the description in terms of Schur polynomials and the eigenvalue basis coincide. With this information we reduce the problem of three basis to two, and the relation between them is very explicit. From our point of view, this is an exact open-closed string duality and this result should be viewed as establishing a very natural setup for the AdS/CFT correspondence.

5. Relations to $\mathcal{N} = 4$ SYM

We now want to see that the matrix model we have been studying appears as a decoupling limit of $\mathcal{N} = 4$ SYM theory. This will provide further evidence that this model has a string theory interpretation.

Consider a time slicing of $AdS_5 \times S^5$, similar to the ideas in [23], so that the hamiltonian is given by

$$H_\epsilon = \frac{(\Delta - J) + \epsilon\Delta}{\epsilon} \tag{5.1}$$

where Δ is the dilatation operator, and J is one of the R-charges of $\mathcal{N} = 4$ SYM.

Take now $\lim_{\epsilon\to 0} H_\epsilon$ and we find that for any state where $\Delta - J > 0$[2] the hamiltonian gives a very large energy, so these states can be decoupled from the low energy theory. The only states which remain are the half BPS states of $AdS_5 \times S^5$. It can be easily seen that in the free field theory limit of $\mathcal{N} = 4$ SYM theory, all other states which are not half BPS with respect to the R-charge J have $\Delta - J \geq 1$, so that they carry very large energy with respect to H. Even in the presence of interactions we expect that these states will not suddenly become very light, so that a decoupled sector remains, as the low-lying states are protected by supersymmetry and are not lifted from having zero energy.

We will now argue that the description of this limit gives exactly a one matrix quantum mechanical system (the matrix will be complex, but it is characterized by the same number of states as we have been discussing in the rest of the paper.)

[2]The BPS inequality is $\Delta - J \geq 0$

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The first thing we need to establish is a way of comparing the hermitian matrix model results with BPS operators in $\mathcal{N}=4$ in SYM theory. The correspondence proceeds as follows. Take SYM on S^3 and choose the gauge in the classical vacuum so that $A=0$. Now, we decompose all fields in their spherical harmonics. For the complex scalars $(\phi^i)^j_k$ one has a singlet under the SO(4) symmetry group, which is a constant mode on the S^3, this spherical harmonic is ϕ^i_0. This corresponds to the local operator $\phi^i_j(0)$ in the operator state-correspondence. The other spherical harmonics are states which transform non-trivially under the SO(4) of rotations, and these are given in the local operator language by covariant derivatives of the field $(\phi^i_j)_\alpha \sim D_\alpha(\phi^i_j)$, where we think of D_α as a derivative operator of order $|\alpha|$ depending on the appropriate representation of SO(3) which is completely symmetric in the derivatives (there is after all a non-trivial commutation relation between covariant derivatives which leads to ambiguities of "normal ordering" of the derivatives.) This definition has the correct free field theory limit.

In this free field limit, the quantization produces one harmonic oscillator per mode on the sphere S^3. The complex field for a given spherical harmonic will be a linear combination of creation operators for the field ϕ^i and annihilation operators for the field $\bar{\phi}^i$. The dictionary then states that

$$\phi^i_\alpha \sim Y_\alpha (a^i)^\dagger_\alpha + Y_\alpha (\bar{a}^i)_\alpha \,. \tag{5.2}$$

We will consider only half BPS operators where the SO(6) R-symmetry is broken down to SO(4). This is, we will be interested in half BPS states which are highest weights of SO(6). These are operators that depend on only one complex scalar of the $\mathcal{N}=4$ multiplet, let us call it ϕ.

When we take the operator $\mathrm{tr}(\phi^n)(0)$ or $\mathrm{tr}_R(\phi)(0)$, which is a half-BPS state, we are instructed to take the state $\mathrm{tr}((\phi_0)^n)|0\rangle$ or $\mathrm{tr}_R(\phi_0)|0\rangle$ on S^3, where we are restricted to the S-wave on the S^3. Notice that because these states are made of spherically invariant oscillators, we do not need to worry about the gauge fields which are non-spherically symmetric. The only spherically symmetric gauge field is the S-wave of A_0. This field is non-dynamical, but it also survives the limit and is required to implement the gauge invariance constraint on the allowed sates coming from the spectrum of $\mathcal{N}=4$ SYM theory.

From the $\mathcal{N}=4$SYM theory, when we choose the different time slicing, we just keep the creation operators for quanta of ϕ and not $\bar{\phi}$, so we get the same hamiltonian as we studied. As argued above, all other states become very massive, so we can ignore them. Even in the interacting field theory case there are no higher polynomial interactions of the field ϕ that do not involve $\bar{\phi}$ or other fields: all of these are set to zero because we are on the ground state for the $\bar{\phi}$ oscillators. The description of the half BPS states then requires us to choose the matrix model with harmonic oscillator potential.

The reader might complain that the matrix ϕ is complex and the U(N) gauging is not sufficient to diagonalize it. However, in supersymmetric field theories the gauge group is usually complexified. Also, we are only keeping half of the degrees of freedom of the complex matrix pair $\phi, \bar{\phi}$ so in the end we have the same number of dynamical degrees of freedom as the model studied in this paper. At least formally, this produces a decoupled sector of the AdS/CFT correspondence which is consistent, as all other degrees of freedom are integrated out because they cost too much energy.

A description in terms of a complex matrix model is also natural if one views the eigenvalue phase space as the description of the lowest Landau level of a 2D fermion in a magnetic field. In this case the coordinates X, Y describing the degeneracy of the landau levels do not commute, and can be associated to the phase space of a single coordinate X. This is how we can relate the model to the quantum Hall effect. Indeed, recently it has been argued in [42] that how one chooses to order the states in the 2D system is analogous to choosing a time coordinate in general relativity. Taking $Z = X + iY$ and $\bar{Z} = X - iY$ as the phase space coordinates produces a complex matrix model with a single complex coordinate Z, which is clearly equivalent to the matrix model for X alone.

Since the annihilation operators $\bar{a}$ act trivially on the vacuum, and since for the complex scalar field there are no self-contractions, the gauge invariant states that we can build with finite energy are just of the form that we described in sections 2 and 4, namely either traces or Schur polynomials of a unique matrix creation operator acting on the vacuum. In this way, we should be able to interpret the states created by Schur polynomials in terms of the eigenvalues of the complex matrix ϕ.

Of course, the study of half-BPS objects is interesting only if there are nice configurations which can be interpreted geometrically on $AdS_5 \times S^5$ and we want to understand the AdS/CFT dictionary. Such objects exist, and they have an interpretation as dynamically stable D-brane solutions: giant gravitons. Giant gravitons [31] are D-brane solutions found in $AdS_5 \times S^5$ which wrap an S^3 and spin on S^5 and which preserve half of the supersymmetries. Hence they have the same quantum numbers as gravitons. These were used to explain the stringy exclusion principle: in this case, the fact that there is an upper bound on the angular momentum of a single string state, namely $\mathrm{tr}(\phi^N)$. Later it was found that there are other D-brane objects, giant gravitons which expand on AdS and which are also spinning on the S^5 which have the same quantum numbers [32, 33] where there is no upper bound on the angular momentum that they carry. In the dual $\mathcal{N} = 4$ theory, they should be represented by some local operator. In the papers [34] and [26] it was proposed that there are two types of operators which correspond to having giant gravitons in the dual spacetime. The two operators which were conjectured to be dual to these two D-brane configurations are given by $\mathrm{tr}(\phi)_R$ for two very simple Young tableaux: the ones with one column (totally antisymmetric representations of $GL(N, \mathbb{C})$ [34]) or the ones with one row (totally symmetric representations of $GL(N, \mathbb{C})$ [26]). Their evidence for these operators corresponding to D-branes was that operators made of traces mix too much to be useful, so operators with better orthogonality properties should do the trick.

Now, let us turn to the matrix model and see what these operators do in the eigenvalue basis. An operator which is totally symmetric with a number of boxes m of order N is identified with the state $\mathrm{tr}_R(a^\dagger)$, where R is the associated Young tableaux to the totally symmetric representation: one row of boxes. We previously saw that this description in terms of Young tableaux corresponds exactly to the eigenvalues basis. Thus, a totally symmetric representation corresponds to taking the topmost eigenvalue of the Fermi sea and giving it an additional energy m which is large. This configuration is thus an eigen-

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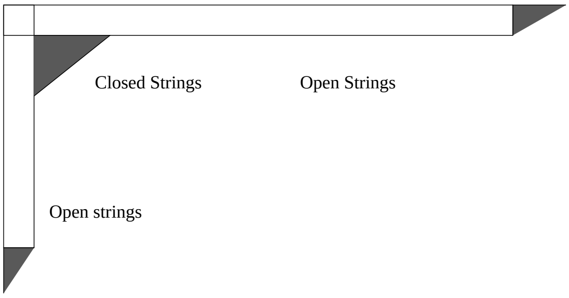

Figure 3: Young diagram identifications of open and closed strings.

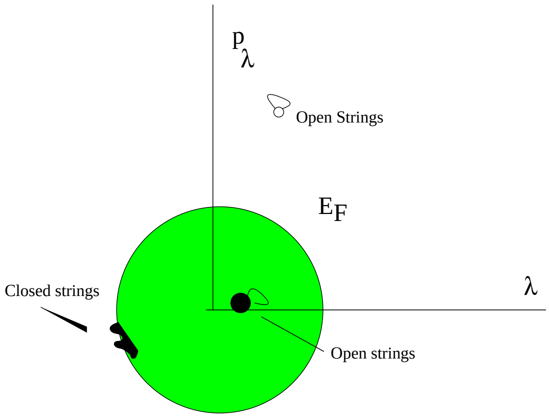

Figure 4: The interpretation of the Young diagram strings in the eigenvalue phase space.

value very far from the Fermi sea. This is the picture of D-branes found in the $c = 1$ matrix model [1, 2]. This is a very natural description also in light of the ideas for Matrix theory in [35]. The original description as an "operator" state makes it look like a giant graviton can only be understood in a very "quantum mechanical" way in terms of the theory on the boundary. This shows that there is a way to think about this D-branes in a more traditional sense. The second type of operator, a giant graviton expanding on S^5 corresponds to a Young tableaux which is a column with m boxes, and $m \leq N$ but of order N. This corresponds to taking the m topmost eigenvalues and giving to each one quantum of energy. This creates a hole deep in the Fermi sea of eigenvalues of the matrix model, which corresponds to another type of D-brane state which was also described in the $c = 1$ matrix model and the boundary Liouville field theory in [7, 36]. So from the $c = 1$ matrix model perspective it is natural that these two objects behave like D-branes. In principle we could have started in the opposite direction and discovered the giant graviton operators.

One should contrast this intuition with the much more cumbersome combinatorial techniques that were used in [37, 38] to show that these states have a well defined $1/N$ expansion and can accommodate a spectrum of open strings. However, some of these results went beyond the study of half BPS operators alone. At least if we restrict to half BPS objects, we can identify states easily in the Young diagrams that correspond to open string excitations and closed string excitations very explicitly. See the figures 3 and 4.

The physical description is as follows: to change the position of the lone eigenvalue or the hole we add or subtract a finite number of boxes from the column or row. These operations are interpreted as exciting the open strings from the D-brane to itself. To add small excitations to the Fermi sea we act on the topmost eigenvalues of the Fermi sea and add some boxes.

Here, from the matrix model point of view the stringy exclusion principle has a different interpretation: the Fermi sea is not infinitely deep. Thus holes in the Fermi sea have a bound on their energy. These are the giant gravitons that expand into S^5.

Also, the Young tableaux lets us visualize the gauge symmetry enhancement when two D-branes come together. This is shown schematically in figure 3 where the open string excitations are drawn in triangular form. This is the constraint on Young tableaux from the fact that the rows have non-increasing length. If we start with k equal rows and add boxes, we get that the excitations coincide with the set of gauge invariant operators for a U(k) gaussian matrix quantum mechanics. The same is true for the holes. If on the other hand we have two D-branes with very different energies, we can add excitations to each of them independently of the other one. This can be interpreted as the Higgs mechanism, where the U(2) gauged symmetry is broken to $U(1)^2$ and the D-branes become very separated from each other. Then we only have fluctuations of the positions of each D-brane independently of the other one.

The new point of view on giant gravitons also helps in defining a new way to do calculations with giant gravitons: a semiclassical calculation. This is in line with the observation of [39] that any time that a large quantum number appears, the result can be understood in terms of semiclassical physics. In this case, it is the semiclassical physics of the single eigenvalue which generates the giant graviton. Based on the AdS dual description of the giants it was conjectured that this was the case [33] because the RR flux inside the giant was reduced by one.

The simplest case is that of a single giant graviton, when we excite one eigenvalue very much. We need to keep track of both the field $\phi_0 = X + iY$ and $\bar{\phi}_0 = X - iY$ to make this semiclassical calculation explicit. Indeed, in terms of X, Y the effective hamiltonian for the free field theory limit is

$$\frac{1}{2}(\mathrm{tr}(p_X^2 + X^2 + p_Y^2 + Y^2)) \tag{5.3}$$

while the R-charge angular momentum is $J = \mathrm{tr}(p_x Y - X p_y)$. The general classical solution for one excited eigenvalue of X,Y is

$$\begin{aligned} X &= \mathrm{diag}(\lambda, 0, 0, \dots, 0)\sin(t+\varphi) &(5.4)\\ Y &= \mathrm{diag}(\lambda', 0, 0, \dots)\sin(t+\chi) &(5.5)\\ p_X &= \mathrm{diag}(\lambda, 0, 0, \dots, 0)\cos(t+\varphi) &(5.6)\\ p_Y &= \mathrm{diag}(\lambda', 0, 0, \dots)\cos(t+\chi) &(5.7)\end{aligned}$$

while the BPS constraint is $J = H$. The classical value of H is given by

$$H = \frac{1}{2}(\lambda^2 + \lambda'^2) \tag{5.8}$$

and

$$J = (\lambda\lambda')\sin(\chi - \varphi) \tag{5.9}$$

The BPS condition becomes $\varphi = \chi - \pi/2$ and $\lambda = \lambda'$. This means that ϕ only has a positive component frequency: $\phi = \mathrm{diag}(\lambda, 0 \dots 0)\exp(it + \varphi)$.

In principle, one should be able to use this solution in the interacting theory to obtain some information on the spectrum of open strings. The D-branes realized by holes should also have a semiclassical description, but we need to treat the hole very differently from the eigenvalue above. In principle, we should aim to match results available in the literature [40, 38], once a framework for doing the semiclassical calculation is found.

6. Other interesting features

The picture of eigenvalues in the Fermi surface also has interesting consequences for the string perturbation expansion. Indeed, we can ask what is the physical reason that in the BMN limit of $\mathcal{N}=4$ SYM [23], the non-planar perturbation expansion for overlap terms in terms of traces behaves like an expansion in J^4/N^2 [22, 24, 25] with that particular power of J. The $1/N^2$ dependence is exactly the one expected from 't Hooft.

From the eigenvalue picture, the wave functions of eigenvalues near the top of the Fermi sea behave roughly as $\lambda^n \exp(-\lambda^2/2)$, and they have a maximum for $\lambda \sim \sqrt{n}$. Expanding around the maximum we find that the value of the wave function decays for large λ as $\exp\left(-\frac{1}{2}(\lambda-\sqrt{n})^2\right)\psi_n(\lambda_0)$, so that the thickness of the wave function for each eigenvalue is more or less uniform, of size 1. The string states with momentum J will create perturbations of the Fermi sea with period J around the circle. One can expect that the continuum description in terms of collective behavior of the eigenvalues will start to break down when we can resolve the individual eigenvalues near the top of the Fermi sea. If we choose some cutoff thickness near the top of the sea, which is given by the thickness of the wave functions, (namely of order 1), then the number of eigenvalues at the top is of order $\sqrt{N}$. This means that for string states of angular momentum of order $J \sim \sqrt{N}$ the perturbation theory in $1/N^2$ should start to converge badly, because we can not ignore the granularity of the eigenvalues beyond this point. This is exactly what is observed in the direct calculations, and resonates with the observations of Shenker on the nature of non-perturbative effects in matrix models being always related to the dynamics of a single eigenvalue [41]. Indeed, adding one eigenvalue costs N in energy for the ground state. This is of order $1/g_s \sim 1/(1/N)$, as expected for the tension of a D-brane.

We can also ask if this matrix quantum mechanical model arises naturally in some other string theory than $\mathcal{N}=4$ SYM theory. We understand this intuition for the $c=1$ matrix model [1], so it would be nice to have a similar description of the setup presented in this paper. One could conjecture that there are solutions of string theory where one can place D0-branes in a potential well where there is one almost massless modulus for the D-branes, and there is no need to require the background where the D-branes are located to be supersymmetric. The low energy limit of the open string theory on these D-branes would look as a gauged matrix quantum mechanics with some potential along this flat direction, which would begin with a quadratic term. Then we could hope that the near horizon geometry of these D0-branes would be holographically dual to the gauged matrix model above.

For example, Verlinde [19] has shown that in a supersymmetric gauged conformal quantum mechanics there is a time slicing of AdS_2 which leads to a quadratic potential for eigenvalues plus potential terms that relate different eigenvalues and depend on some fermion occupation numbers. These extra potential terms in the eigenvalues that can be turned off by requiring the fermionic components of the supereigenvalues to be in their ground states (set $\kappa_{ij} = 1$ in equation 55 of his paper). This sector coincides with the matrix model studied in this paper, and it seems to arise from a completely different physical system which does live in $1 + 1$ dimensions.

7. Conclusion

We have argued in this paper that the gauged gaussian matrix quantum mechanics has the potential to describe a string theory which is exactly solvable, much in the spirit of the $c = 1$ matrix model, and which behaves very much like the AdS/CFt correspondence.

We have presented strong evidence for this claim: we have shown that in this model we can explicitly describe the open-closed string duality, by relating the eigenvalues of the matrix model to the closed string states made out of traces of the fields. This depends on a gauge choice of how we choose to solve the model. Moreover, we have also shown that the system arises as a decoupling limit of $\mathcal{N} = 4$ SYM theory, by taking a different time slicing of the associated AdS spacetime. From this point of view we have been able to show that the eigenvalues of the matrix model behave as D-branes, as well as holes in the Fermi sea of eigenvalues. This has been motivated by a comparison between this model and the operators which describe half-BPS giant gravitons in $AdS_5 \times S^5$. Also, this comparison shows that the giant gravitons have an interpretation in terms of the dynamics of a single eigenvalue of the dual SYM theory. This leaves us with a possibility to study the giant semiclassically both in the supergravity and the SYM theory. Previously, the semiclassical description in SYM was missing (although this description was hinted at in [33]), so at least from this point of view we have learned a valuable piece of information. We have also given some new interpretations to the structure of the non-planar J^2/N expansion in the plane wave limit. We have argued that this dependence on J is natural if we think of it as arising from the breakdown of the collective behavior of the eigenvalues, due to the granularity of the edge of the Fermi sea of the matrix model.

We do not claim to have a geometric dual description because the dual geometry seems to be too strongly curved to provide a semiclassical picture. This in itself should not worry us too much because we have found other examples in string theory, like Gepner models, where a clear interpretation of the geometry is missing. In those cases we are still content with the explicit solution of the string spectrum, and are perfectly happy to call the space time a stringy geometry. Similarly, here we can be content with the same type of description. Based on minimal assumptions, we have guessed that the dual geometry is two dimensional with an AdS like boundary, but beyond that we do not have more information on the geometry.

It would be very interesting to understand how this matrix model is related to the $c = 1$ matrix model in more detail. Naively, the change in the potential from $-X^2/\alpha'$ to X^2/α' looks like an analytic continuation where $\alpha' \to -\alpha'$. However, here the only limit we have is t' Hooft's large-N limit and there is no double scaling limit. The number of eigenvalues on both theories is very different, in the $c = 1$ theory we take $N \to \infty$ strictly, here N

is finite but very large. Indeed, N being fininte was very important to the description of certain aspects of the D-brane physics in terms of Young tableaux.

Probably the most important open problem for this model is to find a target space geometry for the string theory that describes it. Seeing as it arises from a decoupling limit of string theory in $AdS_5 \times S^5$, one suspects that there should be a geometrical description that captures this limit and no additional states.

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